

Title	Eulerノ彈道分弧計算ニ於ケル誤差
Author(s)	渡辺, 次雄
Citation	全国紙上数学談話会. 231 p.768-p.773
Issue Date	1942-01-31
oaire:version	VoR
URL	<a href="https://doi.org/10.18910/74933">https://doi.org/10.18910/74933</a>
rights	
Note	

*Osaka University Knowledge Archive : OUKA*

<https://ir.library.osaka-u.ac.jp/>

Osaka University

# 1003. Eulerノ彈道分弧計算ニ於ケル誤差

渡辺 次 雄(滿洲國陸軍士官學校)

原点ヲ砲口ニ、射面ヲ  $x, y$  平面ニトリシ、 $y$  軸ヲ夫々  
水平及鉛直方向ニトレバ彈道方程式ハ次ノ式ヲ與ヘラ  
レル。

$$\begin{cases} \frac{dv_x}{dt} = -cf(v) \cos \theta & \text{----- (1)} \\ \frac{dv_y}{dt} = -cf(v) \sin \theta - g & \text{----- (2)} \end{cases}$$

但シ空氣抵抗ハ彈丸ノ速度ニノミ關係スルモノト假定スル。  
又  $\theta$  ハ彈道上ノ一点  $(x, y)$  ニ於ケル彈道切線ノ傾斜  
角ヲアル。

$$\frac{dv_x}{dt} = \frac{dv_x}{d\theta} \frac{d\theta}{dt} = -\frac{g \cos \theta}{v} \frac{dv_x}{d\theta}$$

コレヲ (1) = 代入シテ

$$g \frac{dv_x}{d\theta} = v \cdot cf(v)$$

特ニ  $f(v) = v^2 + \text{一定}$  に入る

$$g d(v \cos \theta) = cv^2 = c(v \cos \theta)^3 \frac{d\theta}{\cos^3 \theta}$$

$$(v_x = v \cos \theta)$$

$$\therefore \frac{d(v \cos \theta)}{(v \cos \theta)^3} = \frac{c}{g} \frac{d\theta}{\cos^3 \theta}$$

積分シテ

$$\frac{1}{(v \cos \theta)^2} = -\frac{2c}{g} \int \frac{d\theta}{\cos^3 \theta} + \text{Int-Konst}$$

而シテ

$$\int_0^\theta \frac{d\theta}{\cos^3 \theta} = \frac{\sin \theta}{2 \cos^2 \theta} + \frac{1}{2} \log \tan \left( \frac{\pi}{4} + \frac{\theta}{2} \right)$$

コレヲ  $\xi(\theta)$  デ表ハセバ

$$\frac{1}{(v \cos \theta)^2} = -\frac{2c}{g} \xi(\theta) + \text{Konst.}$$

コノ定数ヲ  $\frac{2c}{g} C$  トカケバ  $C = \frac{g}{2c v_s^2}$  ( $v_s$  ハ彈道頂

点ニ於ケル速度)

$$\therefore \frac{1}{(v \cos \theta)^2} = \frac{2c}{g} (C - \xi(\theta))$$

$$\therefore v^2 = \frac{g}{2c} \frac{1}{\cos^2 \theta (C - \xi(\theta))}$$

$$\text{而 } v = g ds = -v^2 \frac{d\theta}{\cos \theta}$$

$$\therefore 2c ds = - \frac{d\theta}{\cos^3 \theta (C - \xi(\theta))}$$

$$\frac{d\xi}{d\theta} = \frac{1}{\cos^3 \theta} \text{ + ルコトヲ用ヒテ変形スルバ}$$

$$2c ds = - \frac{d\xi}{C - \xi(\theta)} = + \frac{d(C - \xi(\theta))}{C - \xi(\theta)}$$

$$\therefore 2c s = \log(C - \xi(\theta)) + k (\text{Int - Konst.})$$

弧  $s$  ヲ彈道ノ頂点カラ測ルコト = スルバ

$$k = -\log C \quad (\because s=0 \text{ , } \theta=0)$$

$$\therefore s = \frac{1}{2c} \log \frac{C - \xi(\theta)}{C}$$

故 = 圖 = 於テ

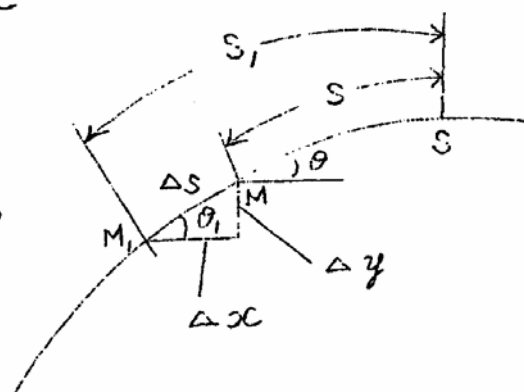
$$SM = s = \frac{1}{2c} \log \frac{C - \xi(\theta)}{C},$$

$$SM_1 = s_1$$

$$= \frac{1}{2c} \log \frac{C - \xi(\theta_1)}{C}$$

$$\therefore \widehat{MM_1} = \Delta s = \frac{1}{2c} \log \frac{C - \xi(\theta_1)}{C - \xi(\theta)}$$

$$\therefore \Delta x = \frac{1}{2c} \log \frac{C - \xi(\theta_1)}{C - \xi(\theta)} \cos \frac{\theta + \theta_1}{2}$$



$$\Delta y = \frac{1}{2C} \log \frac{C - \xi(\theta_1)}{C - \xi(\theta)} \sin \frac{\theta + \theta_1}{2}$$

ナル近似式ヲトルト $x, y$ ハ次ノ式ヲ計算サレル。

$$x = \sum \Delta x = \frac{1}{2C} \sum \log \frac{C - \xi(\theta_1)}{C - \xi(\theta)} \cos \frac{\theta + \theta_1}{2}$$

$$y = \sum \Delta y = \frac{1}{2C} \sum \log \frac{C - \xi(\theta_1)}{C - \xi(\theta)} \sin \frac{\theta + \theta_1}{2}$$

コレハ1753年 L. Euler = ヨツテ與ヘラレタ (Berl. Berl. 1753, S. 348)

本談話ハコノ分弧計算ノ誤差評価式ヲ求メタリ。

明ラカニ

$$\cos \theta_1 < \cos \frac{\theta + \theta_1}{2} < \cos \theta$$

$$\therefore \sum \log \frac{C - \xi(\theta_1)}{C - \xi(\theta)} \cos \theta_1 < 2Cx$$

$$< \sum \log \frac{C - \xi(\theta_1)}{C - \xi(\theta)} \cos \theta$$

故ニ全彈道ニツイテ  $2Cx$  ノ計算カラ生ズル誤差ヲ  $2C\varepsilon$  トスレバ

$$2C\varepsilon = \sum \log \frac{C - \xi(\theta)}{C - \xi(\theta)} \cos \theta - \sum \log \frac{C - \xi(\theta_1)}{C - \xi(\theta)} \cos \theta_1$$

$$= \sum \log \frac{C - \xi(\theta)}{C - \xi(\theta)} \cdot 2 \sin \frac{\theta + \theta_1}{2} \sin \frac{\theta_1 - \theta}{2}$$

落角ヲ  $\omega$  ( $< \frac{\pi}{2}$ ) トスレバ落角ハ射角ヨリ大ナルコトヨリ

$$\left\langle \sum \log \frac{C - \xi(\theta)}{C - \xi(\theta)} \cdot 2 \sin \omega \sin \frac{\Delta \theta}{2} \right\rangle$$

故 = 彈道曲線ノ長サヲ  $S$  トスレバ

$$\begin{aligned} 2C\varepsilon &< 2CS \cdot 2 \sin \omega \cdot \sin \frac{\Delta \theta}{2} \\ &= 4C \sin \omega \sin \frac{\Delta \theta}{2} \cdot S \end{aligned}$$

同ジ射角  $\alpha$ , 同ジ初速  $v_0$  / 真空彈道ノ長サヲ  $S'$  トスレバ  $S' > S$ .

$$\text{且ツ } S' = \frac{2v_0^2}{g} \cos^2 \alpha \int_0^\alpha \frac{d\theta}{\cos^3 \theta} = \frac{2v_0^2}{g} \cos^2 \alpha \cdot \xi(\alpha)$$

$$\therefore \boxed{2C\varepsilon < \frac{8Cv_0^2}{g} \cdot \sin \omega \cdot \cos^2 \alpha \cdot \sin \frac{\Delta \theta}{2} \xi(\alpha)}$$

例  $\alpha = 40^\circ$ ,  $\omega = 47^\circ 11.3'$ ,  $\frac{Cv_0^2}{g} = 0.517$  トキ  
 $2CX = 0.45$

上ノ不等式ノ右辺ヲ計算スルト約  $1.668 \sin \frac{\Delta \theta}{2}$   
 トナル。故  $= \Delta \theta = 20' =$  トルト

$$2C\varepsilon < 0.0049$$

トナル。  $C$ ハ彈道係数ト称スルモノヲ  $0.010$  乃至  
 $0.00007$ ノ値ヲトル。今  $C = 0.0001$  トスレバ, 射  
 程ハ約  $3750\text{m}$ , 誤差ハ  $49\text{m}$ ヨリ小トナル。』

同様  $= 2Cy$  / 誤差評價式ニ計算出來ル。

$$\boxed{2C\varepsilon < \frac{8Cv_0^2}{g} \cos^2 \alpha \sin \frac{\Delta \theta}{2} \xi(\alpha)}$$

トナル。